



## Capital Projects as Real Options: An Introduction

This note introduces an approach to capital budgeting that relies on option pricing theory to analyze and evaluate capital projects. The approach is intended to supplement, not replace, capital budgeting analyses and investment criteria based on standard discounted cash flow (DCF) methodologies. For a wide range of corporate investments, insights from an options-based analysis can improve estimates of project value and, perhaps more important, enhance project management.

### Motivation

Why treat a corporate investment proposal as a call option, as suggested here, rather than as a bond or an unlevered equity? The latter are easier to understand and value, and the associated analyses are easier to communicate and defend within a large organization. However, many corporate investment proposals, particularly "strategic" ones, bear a stronger resemblance to a call option than to a stock or a bond. Ignoring the option-like features of such projects can lead to poor decisions. The most likely mistakes are: failing to invest in a valuable project because imbedded options are overlooked—this will make the corporation appear shortsighted; and not getting the timing right, that is, committing funds earlier or later than would be ideal.

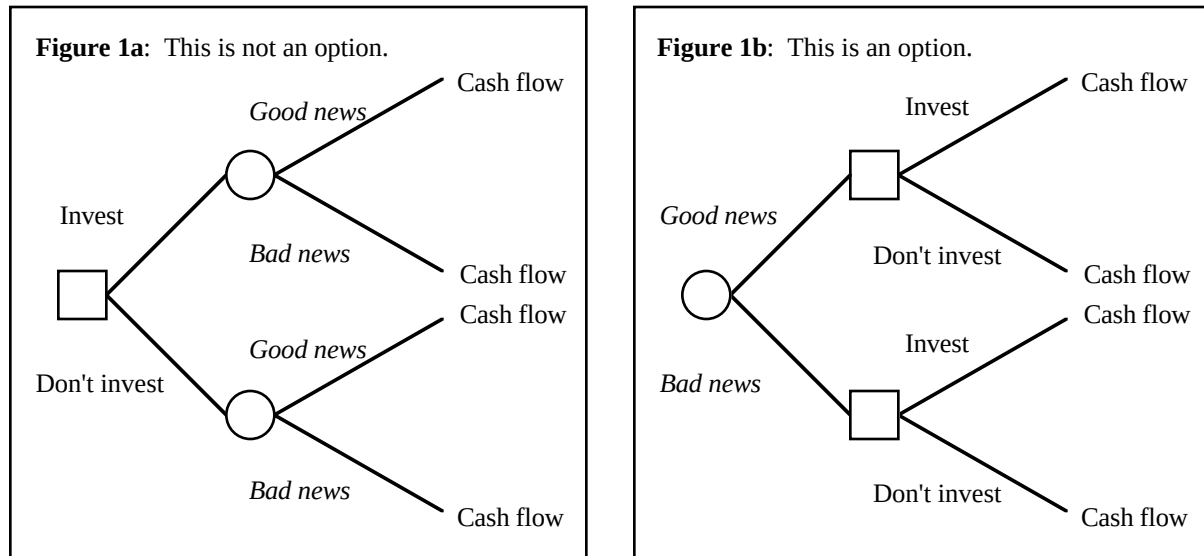
### Asset-in-place vs. options

Standard DCF valuation methodologies treat projects as follows: managers make a decision to invest (or not) and then wait to see what happens (see **Figure 1a**). For some projects this is an adequate representation of reality, but for others it is backwards. Sometimes managers get to wait and see what happens (at least some uncertainty is resolved) and *then* make a decision to invest or not (see **Figure 1b**). These two are obviously quite different. The latter is an option and the former is not. An efficient capital market would not place the same value on both and neither should a corporation.

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*Professor Timothy A. Luehrman prepared this note as the basis for class discussion rather than to illustrate either effective or ineffective handling of an administrative situation.*

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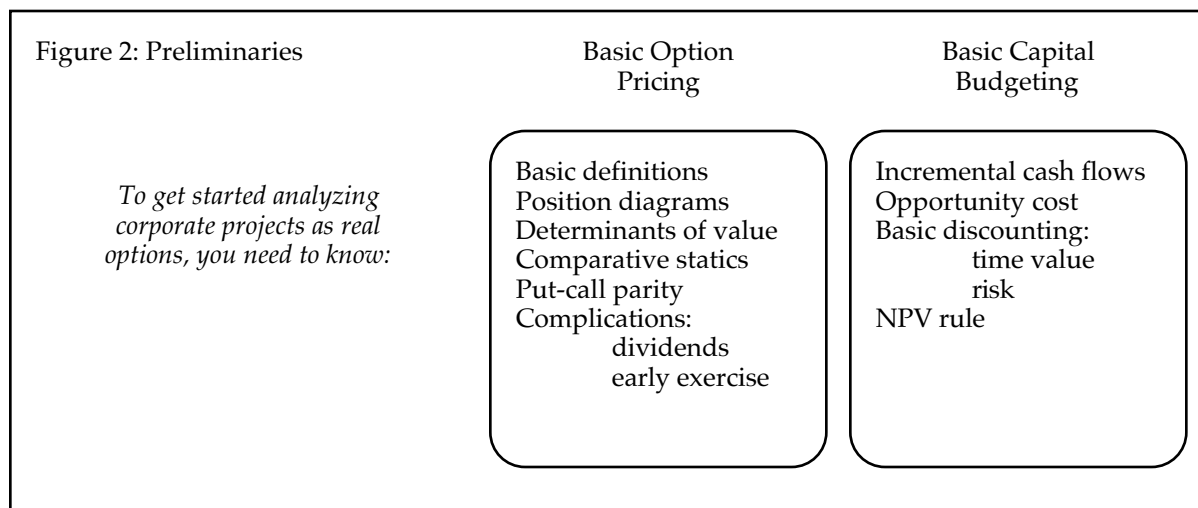
A great many corporate investment proposals are complex and fit neither of these pure archetypes exactly. More often they contain elements of both. An R&D program, for example, may create both a cash-producing new product and opportunities for further R&D aimed at yet more new products. Investing in a new market may lead to both immediate cash flow and future expansion opportunities. Creating a brand may simultaneously create future brand-extension possibilities. Replacing a first-generation technology with a second makes it possible to eventually replace the second with a third, and so forth. All of these examples contain both *assets-in-place* (cash-producing assets that can be evaluated with DCF methodologies) and *growth options* (opportunities to make future investments, which require an option-pricing methodology). Growth options and a few other decision opportunities are known, collectively, as “real” options to distinguish them from “financial” options such as exchange-traded puts and calls. Projects with high option content are likely to be mis-evaluated by DCF techniques: either the options will be ignored (resulting in undervaluation and underinvestment) or they will be poorly approximated (resulting in either under- or over-investment in addition to poor timing and management).

A sensible solution is to separate a project’s assets-in-place from its growth options and to analyze each part accordingly. Unfortunately, this is often difficult, first because a neat separation may not be possible, and second, because executing the option valuation for a capital project is usually a difficult analytical chore. This note is aimed at the second problem. It explains how to set up a mapping between the simplified project and a call option; how to perform an option valuation and relate it to a DCF valuation; and how to extract some managerial insights from the option-pricing framework.

## Preliminaries

As a starting point, this note presumes a working knowledge of basic option pricing and basic capital budgeting. Readers should be acquainted with puts and calls, position diagrams, determinants of call option value, option deltas, and comparative statics. This body of material is

covered in most graduate-level corporate finance texts.<sup>1</sup> Readers also need to be familiar with incremental cash flows, time value, opportunity cost, value additivity, net present value, and the NPV rule.<sup>2</sup>



From this knowledge base, it is possible to construct a mapping between a corporate investment project and a call option and to see the relationship between a project's NPV and the value of an analogous call option.

## Projects as Call Options

An opportunity to invest in a corporate project bears an obvious similarity to an option to invest in a corporation's stock. Both involve the right, but not the obligation, to acquire an asset by paying a certain sum of money on or before a certain time. The right to buy the stock is known as a call option. European calls are exercisable only at expiration, whereas an American call may be exercised at any time prior to expiration. Obviously, an American call must be at least as valuable as an otherwise-identical European call. The Black-Scholes option pricing model gives the value of a European call on a tradable stock that pays no dividends as a function of five variables: the stock price,  $S$ ; the exercise price,  $X$ ; the time to expiration,  $t$ ; the risk-free rate of return,  $r$ ; and the standard deviation of returns on the stock,  $\sigma$ .

By establishing a mapping between project characteristics and the determinants of call-option value, a corporate project can be valued in the same way (see **Figure 3**). Most projects involve making an expenditure to buy or build a productive asset. This is analogous to exercising an option: the amount expended is the exercise price ( $X$ ) and the value of the asset built or acquired is the stock price ( $S$ ). The length of time the company can wait without losing the opportunity is the time to expiration ( $t$ ), and the riskiness of the project is reflected in the standard deviation of returns on the asset ( $\sigma$ ). Time value is still given by the risk-free rate ( $r$ ).

<sup>1</sup> See, for example, Brealey and Myers, *Principles of Corporate Finance*, 4th ed. McGraw-Hill, Inc., New York, 1991, chapter 20, pp. 483-510.

<sup>2</sup> See Brealey and Myers, *ibid.*, chapters 2-6, pp. 11-128.

Figure 3: Mapping Project Characteristics Onto Call Option Variables

| <i>Project</i>                               | <i>Variable</i> | <i>Call Option</i>           |
|----------------------------------------------|-----------------|------------------------------|
| Expenditures required to acquire the assets  | <b>X</b>        | Exercise price               |
| Value of the operating assets to be acquired | <b>S</b>        | Stock price                  |
| Length of time decision may be deferred      | <b>t</b>        | Time to expiration           |
| Riskiness of the underlying operating assets | $\sigma^2$      | Variance of returns on stock |
| Time value of money                          | <b>r</b>        | Risk-free rate of return     |

### Relating DCF Valuation to Option Valuation

To see how option valuation is related to traditional DCF-based capital budgeting, begin by considering the typical project's NPV. NPV is simply a measure of the difference between how much an asset is worth and what it costs. When it is worth more than it costs, the project has a positive NPV and the corporation goes ahead and invests. Put another way,  $NPV = PV(\text{expected net cash flows}) - PV(\text{capital expenditure})$ , and the decision rule is "invest if  $NPV > 0$ ."

Notice that NPV can be expressed as a quotient rather than a difference: Define  $NPV_q \equiv PV(\text{expected net cash flows}) \div PV(\text{capital expenditure})$ . And similarly, the decision rule can be restated as "invest if  $NPV_q > 1$ ." **Figure 4** shows a line on which projects can be arrayed according to  $NPV_q$ . Those for which  $NPV_q > 1$  are accepted; those for which  $NPV_q < 1$  are rejected.

Figure 4: Expressing NPV as a Quotient Rather Than a Difference

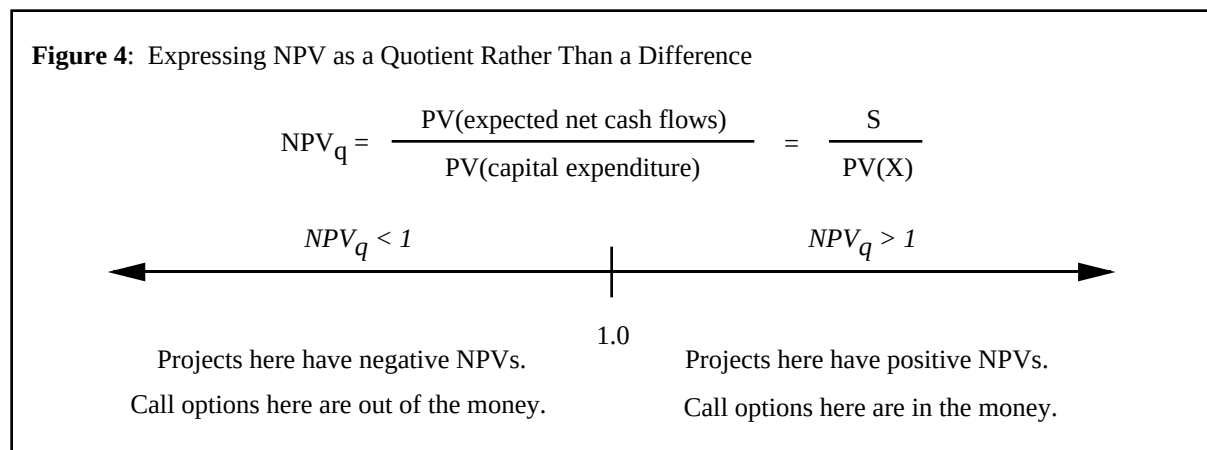


**Figure 4** can be used in the same way to decide whether to exercise a call option *at expiration*. A call option should be exercised if, at expiration, the stock price exceeds the exercise price (the call is "in the money"). Here, the stock price,  $S$ , corresponds to  $PV(\text{expected net cash flows})$  and the exercise price,  $X$ , corresponds to  $PV(\text{capital expenditure})$ . Thus, for a call option,  $NPV_Q = S/PV(X)$ . If this quotient exceeds 1, the option should be exercised. If  $NPV_Q < 1$ , the option is "out of the money" and should not be exercised. In effect, the traditional approach to deciding whether to invest in a project is identical to deciding whether to exercise a call option at expiration. Notice that  $NPV_Q$  combines four of the five determinants of option value:  $S$ ;  $X$ ;  $r$ ; and  $t$ .<sup>3</sup> Note further that call option value is an increasing function of  $NPV_Q$ : the higher  $NPV_Q$ , the higher the call value.

When a decision cannot be delayed, the call option and the project can *both* be evaluated using simple DCF tools and rules—reality is pretty well represented by **Figure 1a**. But when the decision *can* be delayed, the project is like an option that has not yet expired—reality is better represented by **Figure 1b**. In this case,  $NPV_Q$  still matters, but so does the riskiness of the project, which is reflected in the remaining option-pricing variable,  $\sigma$ .

The variability, per unit of time, of returns on the project is measured by the variance of returns,  $\sigma^2$ . Multiplying the variance per unit of time by the amount of time remaining gives *cumulative variance*,  $\sigma^2 t$ . Cumulative variance is a measure of how much things could change before time runs out and a decision must be made. It may be helpful to think of a collection of balls, each with a number on it, that has been placed in an urn. Variance is the amount of variability in the set of numbers written on the balls, and  $t$  is the number of draws to be made from the urn. Cumulative variance is simply the variance for each draw times the number of draws. The more cumulative variance, the more valuable the option.

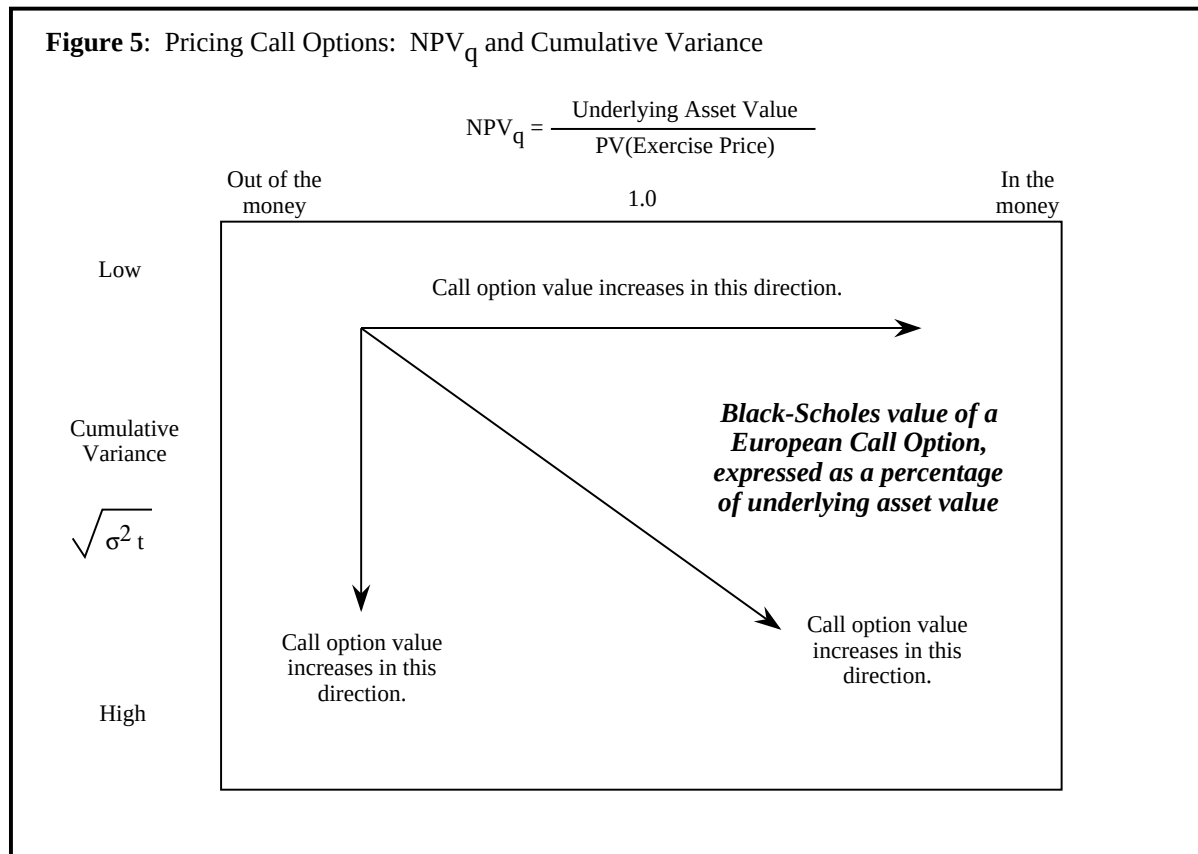
Cumulative variance and  $NPV_Q$  together are sufficient to value a European call option. **Figure 5** expands **Figure 4** to include an extra dimension for  $\sigma\sqrt{t}$ , which is simply the square root of cumulative variance. Options (or projects) for which either  $\sigma$  or  $t$  is zero have no cumulative variance and can be evaluated with standard discounted cash flow techniques, i.e., with NPV or  $NPV_Q$  alone. When both  $\sigma$  and  $t$  are non-zero, however, a DCF analysis will certainly give the wrong value and may lead to the wrong exercise decision: in other words, **Figure 5** should be used rather than **Figure 4**.

The Black-Scholes model, with values for the five variables as inputs, will give a dollar value for a European call option. Some financial calculators and spreadsheet applications come pre-programmed with the Black-Scholes formula. Alternatively, **Exhibit 1** tabulates European call option value as a percentage of the underlying asset value ( $S$ ) for combinations of  $NPV_Q$  and  $\sigma\sqrt{t}$ . The value of a European call can simply be looked up, without a formula or a computer.

To illustrate, consider a simple project that requires an investment of \$100, in return for which the company would receive an asset that is currently worth \$90. However, the asset is risky and its value is likely to change: returns on the asset have a standard deviation of about 40% per year. Moreover, the company can wait for up to three years before deciding to invest. Finally,

<sup>3</sup> The variables  $r$  and  $t$  come into  $NPV_Q$  because  $X$  is being discounted to present value. In the Black-Scholes model, discounting is performed on a continuously-compounded basis, so the present value of  $X$  is actually given by  $X(e^{-rt})$ . Note, though, that at expiration,  $t=0$ , therefore the present value of  $X$  is simply  $X$  and  $NPV_Q$  is simply  $S/X$  as stated in the text. Prior to expiration, however,  $X(e^{-rt}) < X$  so, all else equal,  $NPV_Q > S/X$ .

suppose the risk-free rate is 5%. Viewed conventionally, this project's NPV is  $\$90 - \$100 = -\$10$ . Clearly though, having the opportunity to wait three years and see what happens is valuable. In effect, the company owns a three-year European call with an exercise price of \$100 on underlying assets worth \$90.  $NPV_q$  for this option is  $\$90 \div [\$100 / (1.05)^3] = 1.04$ .<sup>4</sup> Cumulative variance is 0.40 times  $\sqrt{3}$ , or 0.69. **Exhibit 1** shows that an option with these characteristics is worth 28.4% of the value of the underlying asset, or  $.284(\$90) = \$25.56$ .



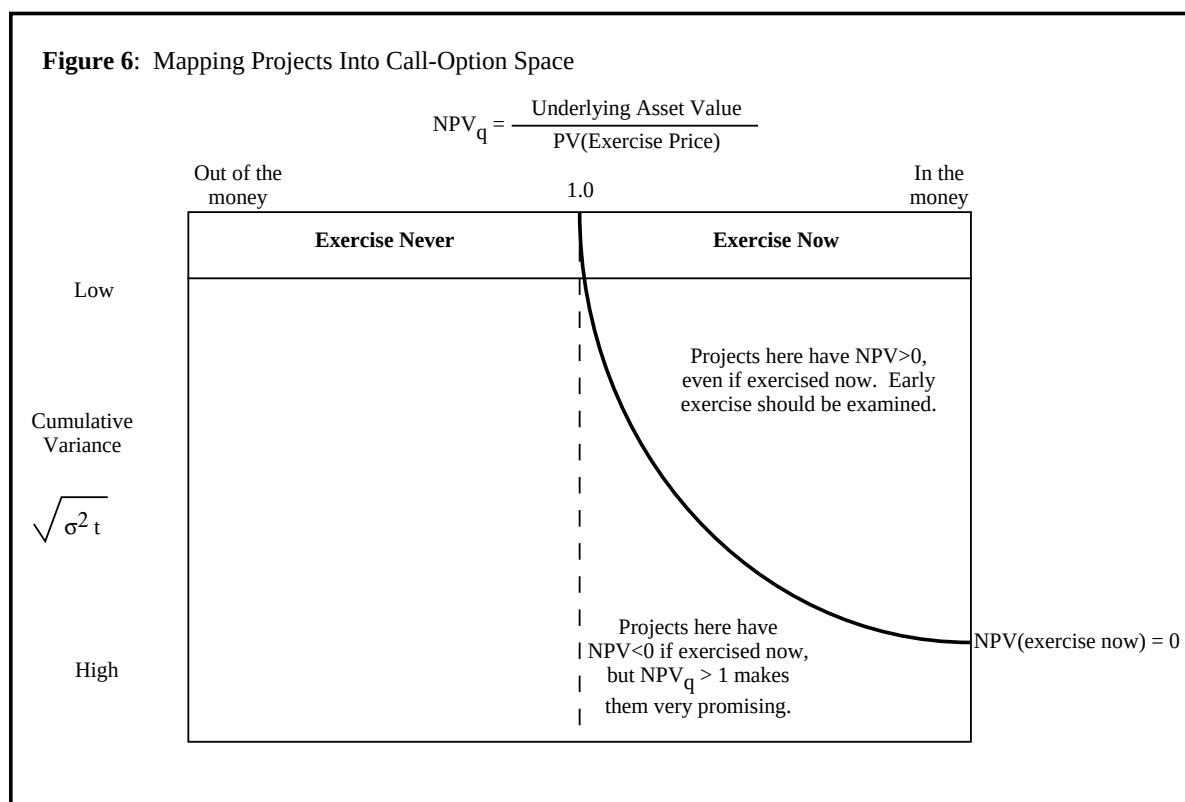
### Managerial Decisions: Optimal Exercise

The simple project just examined had an NPV of  $-\$10$ , but an option value of more than \$25. Are these contradictory? What should the company do? In fact, the NPV and the option value do not contradict one another. The company should not invest in the project now. If it does, it will both forfeit the option and waste \$10. But neither should it discard the project. It should wait, watch, and actively cultivate the project over the next three years. Although  $NPV < 0$ , the project is very promising because  $NPV_q > 1$ . That is to say, although  $X > S$ , these two variables are relatively close to one another because  $S > PV(X)$ . They are separated only by time value. Over time, we expect the

<sup>4</sup> The difference between discrete and continuous compounding to compute the present value of  $X$  is negligible in this case.

market value of the asset,  $S$ , to increase at some rate greater than  $r$  (no one would be willing to hold the asset otherwise).<sup>5</sup> By the end of three years, there is a good chance that the NPV will exceed zero and the option will be exercised. In any event, at expiration, the option will be worth the greater of zero or  $S-X$ . In the meantime, the option on the project really is worth \$25, not -\$10, provided the company does not (suboptimally) exercise it now.

The difference between NPV and  $NPV_q$  contains a useful managerial insight. As time runs out, these two must converge to some agreement: at expiration they will be either greater than 0 and 1, respectively, or less than these values. But prior to expiration,  $NPV_q$  may be positive even when NPV is negative (just as in the preceding example). **Figure 6** shows this diagrammatically. All options that fall in the right half of **Figure 6** have  $NPV_q > 1$ . But not all of these are in the money; that is, the NPV of an "exercise-now" strategy is positive for some and negative for others. The locus of points that corresponds to  $NPV(\text{exercise now})=0$  is a curve that starts at the top, where cumulative variance is zero and  $NPV_q=1$ , and runs down and to the right.<sup>6</sup> Options that fall above this curve have both  $NPV > 0$  and  $NPV_q > 1$ ; they are in the money. Those below the curve have  $NPV_q > 1$  but  $NPV < 0$ ; they are out of the money.



<sup>5</sup> This argument assumes that the asset is like a stock that pays no dividends. The exception to this case is quite important and is treated below.

<sup>6</sup> The location of this curve varies with  $r$  and  $\sigma$ . The curve is located by holding  $r$  and  $\sigma$  constant as  $t$  varies, and solving for the  $NPV_q$  that corresponds to  $NPV=0$ . Note that in the extreme case of  $r=0$ , the curve is a vertical line passing through  $NPV_q=1$ . As  $r$  increases, the slope of the curve decreases, bending to the right, as shown in **Figure 6**.

We can now generate three different managerial prescriptions for options with  $NPV_Q > 1$ , each corresponding to a different region in the right half of **Figure 6**. At the very top are options with no cumulative variance—either time has run out or there is no variance. These options are in the money ( $NPV > 0$ ) and should be exercised immediately—there is no value in waiting.

Just below these are options that are in the money, but for which there is still some cumulative variance. The company should wait, *if possible*, to exercise these options. Early exercise may be desirable when the underlying asset is "wasting." If, for example, its value is subject to erosion due to competitors' actions, or if it is already paying out cash, then it is analogous to a dividend-paying stock. Holders of American call options on a dividend-paying stock sometimes will find it optimal to exercise early, prior to expiration. This is a way to capture the cash being paid out or prevent the value erosion. However, by exercising early, the holder of an American call option foregoes the interest on the exercise price. Hence, a tradeoff must be evaluated in order to determine the better course of action. The distinction between American and European calls is very important for real options. Real options typically *can* be exercised early and often the value of the underlying asset is subject to erosion by competitors' actions or technical or demographic changes.<sup>7</sup>

Finally, at the bottom of the right half of **Figure 6** are options like the simple example presented above: they are very promising because  $NPV_Q > 1$  even though  $NPV < 0$ . If, as time runs out, neither  $S$  nor  $X$  changes, then  $NPV_Q$  will fall and these options will expire unexercised. But among a large sample of such projects, we should expect many to end up in the money, especially if they receive active attention and management.

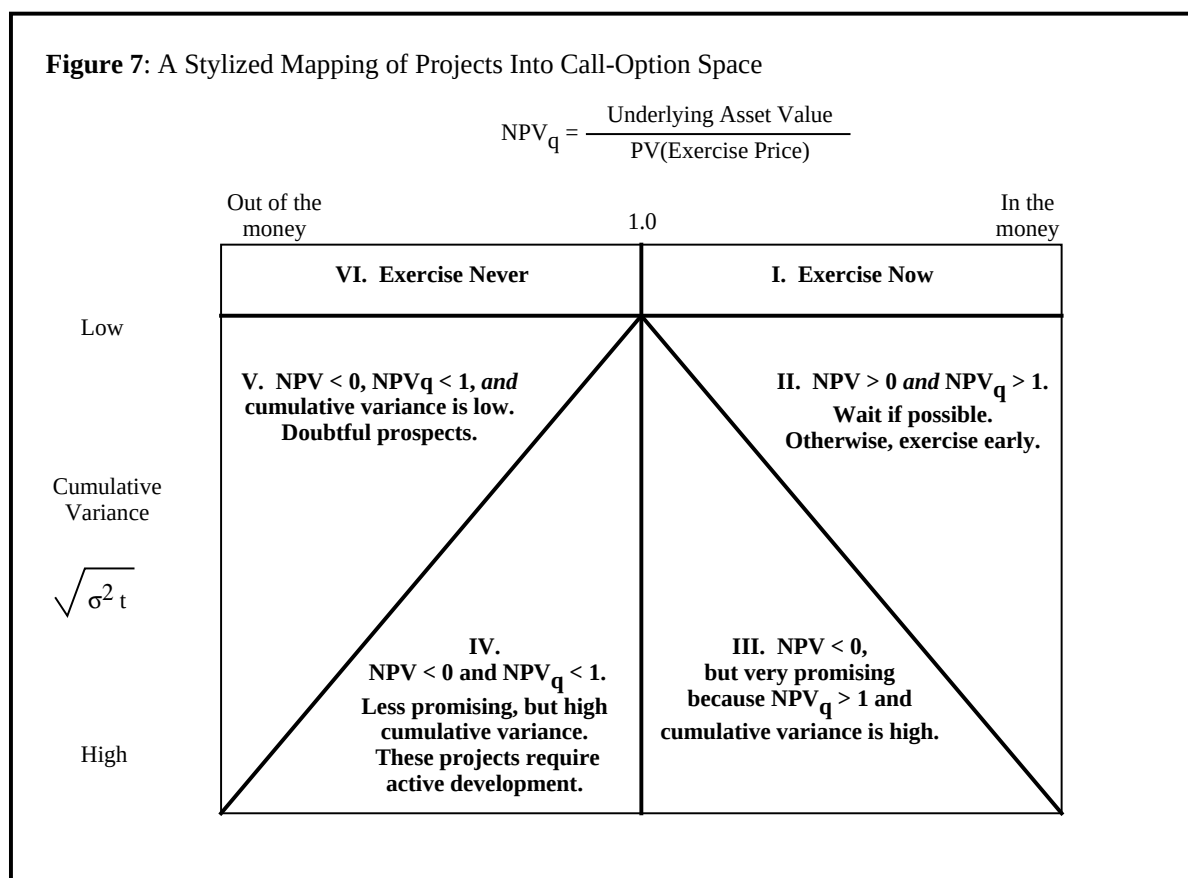
### A Stylized Map: The "Tomato Garden"

Pushing the logic of **Figure 6** a bit further, we can divide the call-option space roughly into six regions, each corresponding to a different managerial prescription as shown in **Figure 7**. The right side of **Figure 7** is divided into regions I, II, and III, in all of which  $NPV_Q > 1$ . These correspond to the three regions in the right half of **Figure 6** just described. The left half of **Figure 7** is divided symmetrically, into regions IV, V, and VI, in all of which  $NPV_Q < 1$ . In region VI at the top, cumulative variance is zero so these options are never exercised. Region V contains relatively unpromising options. For them,  $NPV_Q$  and/or  $\sigma\sqrt{t}$  is low. Not many of these projects will make it, regardless of the attention they may receive. In region IV are options for which either  $NPV_Q$  or  $\sigma\sqrt{t}$  is reasonably high, but the other is low. These projects require active development to end up in the money. In general, projects will tend to move upward in **Figure 7** as time passes, because time runs out and uncertainty is resolved. Managers have two jobs: to try to move projects to the right before time runs out; and to avoid making mistakes in their exercise decisions in the meantime.

The analogy of a tomato garden located in an unpredictable climate may be a helpful mnemonic device. Managers are the gardeners; they do the cultivating and eventually decide which tomatoes to pick and which not to. At one extreme are perfect, ripe tomatoes (region I of **Figure 7**) which should be picked and eaten immediately. At the other extreme are rotten tomatoes (region VI) which should never be picked. In between are many different tomatoes with varying prospects. Those in region II are edible and could be picked, but they would improve with more time on the vine. The gardener will pick them early only if competitors (e.g., squirrels) are likely to get

<sup>7</sup> For more on American calls and dividend-paying stocks, see Brealey and Myers, *op.cit.*, chapter 21, pp. 526-29.

them otherwise. Tomatoes in region III are not edible and should not be picked regardless of competitors. Over time, however, many in region III will ripen and eventually be picked. Region IV contains fruit that is less promising and may not ripen before the season is over. Nevertheless, with more sun or water, fewer weeds, or just good luck, some of these tomatoes may become edible. Finally, region V contains late blossoms and small green tomatoes that have not much chance of growing and ripening before the season ends.



## Practical Issues

Using option pricing models to analyze capital projects presents some practical problems. Comparatively few of these have completely satisfactory solutions; on the other hand, some insight is gained just from formulating and articulating the problems. Still more, perhaps, is available from approximations. To interpret an analysis, it helps to remain aware of whether it represents an exact answer to an approximated (simplified) problem, or an approximate answer to an exact problem. Either may be useful.

**Simplifying complex projects** Real corporate projects, especially long-horizon ones, are complex. They are most often combinations of assets-in-place and options. Further, the options are nested; that is, managers face a *sequence* of serially-dependent choices. It helps to simplify such problems, not only for formal analytical purposes, but also to make them understandable and

discussible within the group of responsible managers. Further, an unsimplified problem may be unique (and practically insoluble), whereas a simplified problem may fit in a larger *class* of problems which will provide hints or boundaries for a solution.

Most real option problems can either be abstracted as fairly simple call options, or broken into pieces, each one of which is a fairly simple call option. A useful guide in simplifying is to search for the primary uncertainty against which managers select. That is, by waiting and watching, what is the main thing a manager learns, and how will he or she exploit the learning? Some projects have option-like characteristics (e.g., the project is risky and managers clearly have a choice about whether to undertake it), but yet as long as the company does nothing, it learns nothing. For such projects, a DCF approach may be appropriate; in effect, the company has to make the investment in order to resolve the uncertainty (such projects resemble **Figure 1a**). However, for many other projects, the resolution of a small set of uncertainties determines the outcome, *and* at least some can be resolved without making the entire investment. Such projects resemble **Figure 1b**, and identifying the key uncertainties is the first step toward a useful simplification.

Another useful approach is to construct simplifications such that the simplified project is both priceable and either dominating or dominated, compared to the real project. A dominated project is one that is clearly *less* valuable than the real project. If it also can be priced, then its value provides a lower bound for the project value. For example, a simple European call is clearly dominated by a (more realistic) American call on an asset exposed to erosion by competitors' investments. Dominating simplifications similarly provide an upper bound for project value. Sometimes it is possible to construct and compute both upper and lower bounds.

**Estimating volatility** The option-pricing input managers are least accustomed to estimating is variance, or standard deviation. For a real option,  $\sigma$  cannot be looked up in a table or newspaper, and most people do not have highly developed intuition about, for example, the annual variance of returns on assets associated with entering a new market. Nevertheless, estimating  $\sigma$  is not a completely hopeless task. There are at least three reasonable approaches:

1. *Take a guess.* Systematic risk ( $\beta$ ) and total risk ( $\sigma$ ) are positively correlated in large samples of operating assets: those with high asset betas are likelier to have higher standard deviations. What is a high standard deviation? Returns on broad-based U.S. stock market indices had a standard deviation of about 20% per year for much of 1986-94, with exceptions (upward spikes) associated with events like the 1987 market crash and the 1990-91 Persian Gulf crisis. Individual projects will have higher volatilities than a diversified portfolio of the same projects. Volatility of 20-30% per year is not remarkably high for a single project.
2. *Gather some data.* Volatility can be estimated for some businesses using historical data on investment returns in certain industries. Alternatively, implied volatilities can be computed from quoted option prices for a very large number of traded equities. The length, breadth, and quality of such data have improved greatly in recent years and should continue to do so. These data do require some adjustment, however. Equity returns are levered and therefore more volatile than underlying asset returns.
3. *Simulate  $\sigma$ .* Spreadsheet-based projections of a project's future cash flows, together with Monte Carlo simulation techniques can be used to synthesize a probability distribution for project returns. Inputs include estimated volatilities for specific items, such as commodity prices or exchange rates. Using these inputs, a computer simulates the project and, in the process, synthesizes a probability distribution for desired output variables, such as project returns. From the synthesized probability distribution,  $\sigma$  can

be estimated.<sup>8</sup> Simulation software for desktop computers is commercially available and designed to work as an add-in with popular spreadsheet applications.

Some insight is available from **Figure 7** even without precise estimates of  $\sigma$ . Simply knowing whether cumulative variance is high or low is enough to locate a project in one part or another of **Figure 7** and, with some knowledge of  $NPV_Q$ , to suggest a managerial course of action.

**Checking models and distributions** All formal option pricing models, including Black-Scholes, assume that the riskiness of an asset can be expressed as a probability distribution for returns (or prices or payouts) for the asset. Some of the assumed distributions are elegantly simple, such as the lognormal distribution assumed by Black-Scholes. But corporate data for some real projects are inelegant and may be inconsistent with, for example, a lognormal distribution. One approach to this problem is to figure out in which direction a simplified distribution biases the analysis and then interpret the output accordingly, as an upper or lower bound for the actual project's value. Another is to choose a model, if one exists, based on a more appropriate distribution. Many models have been constructed and solved, though most are mathematically sophisticated and unwieldy for use with real options.<sup>9</sup>

More fundamental than the particular distribution assumed by a given model is the type of world being modeled. The Black-Scholes world, for example, is one in which underlying assets are securities that are traded continuously. Many real options involve underlying assets that are not traded continuously or, in some cases, not traded at all. For such assets, the five variables (six, if dividends are allowed) of the Black-Scholes model are not sufficient to characterize and price a call option. Whether one model or another remains useful as a way to price a simplified version of the project is a judgment the analyst must make. One alternative to such modeling is brute force, in the form of computing power. High-speed computers and advanced spreadsheet software make it possible to simulate some projects as a complicated decision tree. Decision-tree analysis is not, formally speaking, option pricing, but if well-executed, it provides a better treatment of uncertainty and of managers' scope for decision-making than conventional DCF analysis alone.

**Interpreting results** To execute a useful analysis, simplification is essential. To interpret the results, some sophistication is equally essential. This typically involves layering complexity back into the problem, in the form of sensitivity analyses and the conditioning or qualifying of inferences. To arrive at **Figure 7** and its useful mapping, we first took a complicated project and simplified it enough to regard it as a European call option. Then, drawing on an understanding of option pricing and capital budgeting, we further simplified the analysis by combining five variables into two. But now, after valuing a project and locating it in **Figure 7**, we should put some of the complexity back in and begin looking at sensitivity analyses. This will help us understand which of a project's characteristics cause it to fall where it does on the map. For example, in region III, is cumulative variance high primarily because of  $\sigma$  or  $t$ ? Which of the elements of  $NPV_Q$  is most readily managed—net cash inflows (S) or capital expenditures (X)? And so forth. This process is iterative. The results of one analysis suggest a further one until the process converges on an understanding, in both financial and managerial terms, of the project.

<sup>8</sup> Skillful simulation requires knowledge of probability and statistics, including the forms of distributions, elementary sampling theory, differences between time series and cross-sectional analyses, and so forth. A useful reference is Kelton and Law, *Simulation Modeling and Analysis*, McGraw-Hill, Inc., New York, 1991.

<sup>9</sup> A bibliography of sophisticated models is given in Merton, *Continuous-Time Finance*, Basil Blackwell, Inc., Cambridge, Mass., 1990. A more narrowly-focused reference is Wilmott, Dewynne, and Howison, *Option Pricing: Mathematical Models and Computation*, Oxford Financial Press, Oxford, 1993.

## Exhibit 1 Option-Pricing Table

## Black-Scholes Value of a European Call Option, Expressed as a Percentage of Underlying Asset Value

$$NPVq = (\text{Underlying asset value}) / PV(\text{Exercise price})$$

|      | 0.30 | 0.35 | 0.40 | 0.45 | 0.50 | 0.55 | 0.60 | 0.65 | 0.70 | 0.75 | 0.80 | 0.82 | 0.84 | 0.86 | 0.88 | 0.90 | 0.92 | 0.94 | 0.96 | 0.98 | 1.00 | 1.02 | 1.04 | 1.06 | 1.08 | 1.10 | 1.12 | 1.14 | 1.16 | 1.18 | 1.20 | 1.25 | 1.30 | 1.35 | 1.40 | 1.45 | 1.50 | 1.75 | 2.00 | 2.50 |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       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|        |        |        |        |        |        |        |        |        |        |        |        |        |        |  |
|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------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| 0.05 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.03 | 0.06 | 0.12 | 0.20 | 0.31 | 0.43 | 0.56 | 0.70 | 0.84 | 0.98 | 1.10 | 1.12 | 1.14 | 1.16 | 1.18 | 1.20 | 1.25 | 1.30 | 1.35 | 1.40 | 1.45 | 1.50 | 1.75 | 2.00 | 2.50 |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       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|        |        |        |        |        |        |        |        |        |        |        |        |        |        |  |
| 0.10 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.02 | 0.03 | 0.05 | 0.08 | 0.12 | 0.17 | 0.23 | 0.31 | 0.40 | 0.50 | 0.61 | 0.73 | 0.86 | 1.00 | 1.10 | 1.12 | 1.14 | 1.16 | 1.18 | 1.20 | 1.25 | 1.30 | 1.35 | 1.40 | 1.45 | 1.50 | 1.75 | 2.00 | 2.50 |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       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|       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |  |
| 0.15 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.07 | 0.10 | 0.13 | 0.17 | 0.22 | 0.28 | 0.35 | 0.42 | 0.50 | 0.60 | 0.70 | 0.80 | 0.91 | 1.02 | 1.10 | 1.12 | 1.14 | 1.16 | 1.18 | 1.20 | 1.25 | 1.30 | 1.35 | 1.40 | 1.45 | 1.50 | 1.75 | 2.00 | 2.50 |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |  |
| 0.20 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.04 | 0.08 | 0.15 | 0.19 | 0.23 | 0.28 | 0.34 | 0.40 | 0.47 | 0.54 | 0.62 | 0.71 | 0.80 | 0.89 | 0.99 | 1.09 | 1.19 | 1.28 | 1.37 | 1.44 | 1.50 | 1.56 | 1.63 | 1.74 | 1.85 | 1.92 | 2.00 | 2.08 | 2.16 | 2.24 | 2.32 | 2.40 | 2.50 | 2.60 | 2.70 | 2.80 | 2.90 | 3.00 |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |       |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |        |  |
| 0.25 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.01 | 0.02 | 0.05 | 0.10 | 0.18 | 0.28 | 0.33 | 0.39 | 0.45 | 0.52 | 0.59 | 0.66 | 0.74 | 0.82 | 0.91 | 0.99 | 1.09 | 1.18 | 1.28 | 1.37 | 1.47 | 1.57 | 1.67 | 1.77 | 1.87 | 1.98 | 2.03 | 2.12 | 2.23 | 2.34 | 2.45 | 2.56 | 2.67 | 2.78 | 2.89 | 3.00 | 3.11 | 3.22 | 3.33 | 3.44 | 3.55 | 3.66 | 3.77 | 3.88 | 4.00 | 4.11 | 4.22 | 4.33 | 4.44 | 4.55 | 4.66 | 4.77 | 4.88 | 5.00 | 5.11 | 5.22 | 5.33 | 5.44 | 5.55 | 5.66 | 5.77 | 5.88 | 6.00 | 6.11 | 6.22 | 6.33 | 6.44 | 6.55 | 6.66 | 6.77 | 6.88 | 7.00 | 7.11 | 7.22 | 7.33 | 7.44 | 7.55 | 7.66 | 7.77 | 7.88 | 8.00 | 8.11 | 8.22 | 8.33 | 8.44 | 8.55 | 8.66 | 8.77 | 8.88 | 9.00 | 9.11 | 9.22 | 9.33 | 9.44 | 9.55 | 9.66 | 9.77 | 9.88 | 10.00 | 10.11 | 10.22 | 10.33 | 10.44 | 10.55 | 10.66 | 10.77 | 10.88 | 11.00 | 11.11 | 11.22 | 11.33 | 11.44 | 11.55 | 11.66 | 11.77 | 11.88 | 12.00 | 12.11 | 12.22 | 12.33 | 12.44 | 12.55 | 12.66 | 12.77 | 12.88 | 13.00 | 13.11 | 13.22 | 13.33 | 13.44 | 13.55 | 13.66 | 13.77 | 13.88 | 14.00 | 14.11 | 14.22 | 14.33 | 14.44 | 14.55 | 14.66 | 14.77 | 14.88 | 15.00 | 15.11 | 15.22 | 15.33 | 15.44 | 15.55 | 15.66 | 15.77 | 15.88 | 16.00 | 16.11 | 16.22 | 16.33 | 16.44 | 16.55 | 16.66 | 16.77 | 16.88 | 17.00 | 17.11 | 17.22 | 17.33 | 17.44 | 17.55 | 17.66 | 17.77 | 17.88 | 18.00 | 18.11 | 18.22 | 18.33 | 18.44 | 18.55 | 18.66 | 18.77 | 18.88 | 19.00 | 19.11 | 19.22 | 19.33 | 19.44 | 19.55 | 19.66 | 19.77 | 19.88 | 20.00 | 20.11 | 20.22 | 20.33 | 20.44 | 20.55 | 20.66 | 20.77 | 20.88 | 21.00 | 21.11 | 21.22 | 21.33 | 21.44 | 21.55 | 21.66 | 21.77 | 21.88 | 22.00 | 22.11 | 22.22 | 22.33 | 22.44 | 22.55 | 22.66 | 22.77 | 22.88 | 23.00 | 23.11 | 23.22 | 23.33 | 23.44 | 23.55 | 23.66 | 23.77 | 23.88 | 24.00 | 24.11 | 24.22 | 24.33 | 24.44 | 24.55 | 24.66 | 24.77 | 24.88 | 25.00 | 25.11 | 25.22 | 25.33 | 25.44 | 25.55 | 25.66 | 25.77 | 25.88 | 26.00 | 26.11 | 26.22 | 26.33 | 26.44 | 26.55 | 26.66 | 26.77 | 26.88 | 27.00 | 27.11 | 27.22 | 27.33 | 27.44 | 27.55 | 27.66 | 27.77 | 27.88 | 28.00 | 28.11 | 28.22 | 28.33 | 28.44 | 28.55 | 28.66 | 28.77 | 28.88 | 29.00 | 29.11 | 29.22 | 29.33 | 29.44 | 29.55 | 29.66 | 29.77 | 29.88 | 30.00 | 30.11 | 30.22 | 30.33 | 30.44 | 30.55 | 30.66 | 30.77 | 30.88 | 31.00 | 31.11 | 31.22 | 31.33 | 31.44 | 31.55 | 31.66 | 31.77 | 31.88 | 32.00 | 32.11 | 32.22 | 32.33 | 32.44 | 32.55 | 32.66 | 32.77 | 32.88 | 33.00 | 33.11 | 33.22 | 33.33 | 33.44 | 33.55 | 33.66 | 33.77 | 33.88 | 34.00 | 34.11 | 34.22 | 34.33 | 34.44 | 34.55 | 34.66 | 34.77 | 34.88 | 35.00 | 35.11 | 35.22 | 35.33 | 35.44 | 35.55 | 35.66 | 35.77 | 35.88 | 36.00 | 36.11 | 36.22 | 36.33 | 36.44 | 36.55 | 36.66 | 36.77 | 36.88 | 37.00 | 37.11 | 37.22 | 37.33 | 37.44 | 37.55 | 37.66 | 37.77 | 37.88 | 38.00 | 38.11 | 38.22 | 38.33 | 38.44 | 38.55 | 38.66 | 38.77 | 38.88 | 39.00 | 39.11 | 39.22 | 39.33 | 39.44 | 39.55 | 39.66 | 39.77 | 39.88 | 40.00 | 40.11 | 40.22 | 40.33 | 40.44 | 40.55 | 40.66 | 40.77 | 40.88 | 41.00 | 41.11 | 41.22 | 41.33 | 41.44 | 41.55 | 41.66 | 41.77 | 41.88 | 42.00 | 42.11 | 42.22 | 42.33 | 42.44 | 42.55 | 42.66 | 42.77 | 42.88 | 43.00 | 43.11 | 43.22 | 43.33 | 43.44 | 43.55 | 43.66 | 43.77 | 43.88 | 44.00 | 44.11 | 44.22 | 44.33 | 44.44 | 44.55 | 44.66 | 44.77 | 44.88 | 45.00 | 45.11 | 45.22 | 45.33 | 45.44 | 45.55 | 45.66 | 45.77 | 45.88 | 46.00 | 46.11 | 46.22 | 46.33 | 46.44 | 46.55 | 46.66 | 46.77 | 46.88 | 47.00 | 47.11 | 47.22 | 47.33 | 47.44 | 47.55 | 47.66 | 47.77 | 47.88 | 48.00 | 48.11 | 48.22 | 48.33 | 48.44 | 48.55 | 48.66 | 48.77 | 48.88 | 49.00 | 49.11 | 49.22 | 49.33 | 49.44 | 49.55 | 49.66 | 49.77 | 49.88 | 50.00 | 50.11 | 50.22 | 50.33 | 50.44 | 50.55 | 50.66 | 50.77 | 50.88 | 51.00 | 51.11 | 51.22 | 51.33 | 51.44 | 51.55 | 51.66 | 51.77 | 51.88 | 52.00 | 52.11 | 52.22 | 52.33 | 52.44 | 52.55 | 52.66 | 52.77 | 52.88 | 53.00 | 53.11 | 53.22 | 53.33 | 53.44 | 53.55 | 53.66 | 53.77 | 53.88 | 54.00 | 54.11 | 54.22 | 54.33 | 54.44 | 54.55 | 54.66 | 54.77 | 54.88 | 55.00 | 55.11 | 55.22 | 55.33 | 55.44 | 55.55 | 55.66 | 55.77 | 55.88 | 56.00 | 56.11 | 56.22 | 56.33 | 56.44 | 56.55 | 56.66 | 56.77 | 56.88 | 57.00 | 57.11 | 57.22 | 57.33 | 57.44 | 57.55 | 57.66 | 57.77 | 57.88 | 58.00 | 58.11 | 58.22 | 58.33 | 58.44 | 58.55 | 58.66 | 58.77 | 58.88 | 59.00 | 59.11 | 59.22 | 59.33 | 59.44 | 59.55 | 59.66 | 59.77 | 59.88 | 60.00 | 60.11 | 60.22 | 60.33 | 60.44 | 60.55 | 60.66 | 60.77 | 60.88 | 61.00 | 61.11 | 61.22 | 61.33 | 61.44 | 61.55 | 61.66 | 61.77 | 61.88 | 62.00 | 62.11 | 62.22 | 62.33 | 62.44 | 62.55 | 62.66 | 62.77 | 62.88 | 63.00 | 63.11 | 63.22 | 63.33 | 63.44 | 63.55 | 63.66 | 63.77 | 63.88 | 64.00 | 64.11 | 64.22 | 64.33 | 64.44 | 64.55 | 64.66 | 64.77 | 64.88 | 65.00 | 65.11 | 65.22 | 65.33 | 65.44 | 65.55 | 65.66 | 65.77 | 65.88 | 66.00 | 66.11 | 66.22 | 66.33 | 66.44 | 66.55 | 66.66 | 66.77 | 66.88 | 67.00 | 67.11 | 67.22 | 67.33 | 67.44 | 67.55 | 67.66 | 67.77 | 67.88 | 68.00 | 68.11 | 68.22 | 68.33 | 68.44 | 68.55 | 68.66 | 68.77 | 68.88 | 69.00 | 69.11 | 69.22 | 69.33 | 69.44 | 69.55 | 69.66 | 69.77 | 69.88 | 70.00 | 70.11 | 70.22 | 70.33 | 70.44 | 70.55 | 70.66 | 70.77 | 70.88 | 71.00 | 71.11 | 71.22 | 71.33 | 71.44 | 71.55 | 71.66 | 71.77 | 71.88 | 72.00 | 72.11 | 72.22 | 72.33 | 72.44 | 72.55 | 72.66 | 72.77 | 72.88 | 73.00 | 73.11 | 73.22 | 73.33 | 73.44 | 73.55 | 73.66 | 73.77 | 73.88 | 74.00 | 74.11 | 74.22 | 74.33 | 74.44 | 74.55 | 74.66 | 74.77 | 74.88 | 75.00 | 75.11 | 75.22 | 75.33 | 75.44 | 75.55 | 75.66 | 75.77 | 75.88 | 76.00 | 76.11 | 76.22 | 76.33 | 76.44 | 76.55 | 76.66 | 76.77 | 76.88 | 77.00 | 77.11 | 77.22 | 77.33 | 77.44 | 77.55 | 77.66 | 77.77 | 77.88 | 78.00 | 78.11 | 78.22 | 78.33 | 78.44 | 78.55 | 78.66 | 78.77 | 78.88 | 79.00 | 79.11 | 79.22 | 79.33 | 79.44 | 79.55 | 79.66 | 79.77 | 79.88 | 80.00 | 80.11 | 80.22 | 80.33 | 80.44 | 80.55 | 80.66 | 80.77 | 80.88 | 81.00 | 81.11 | 81.22 | 81.33 | 81.44 | 81.55 | 81.66 | 81.77 | 81.88 | 82.00 | 82.11 | 82.22 | 82.33 | 82.44 | 82.55 | 82.66 | 82.77 | 82.88 | 83.00 | 83.11 | 83.22 | 83.33 | 83.44 | 83.55 | 83.66 | 83.77 | 83.88 | 84.00 | 84.11 | 84.22 | 84.33 | 84.44 | 84.55 | 84.66 | 84.77 | 84.88 | 85.00 | 85.11 | 85.22 | 85.33 | 85.44 | 85.55 | 85.66 | 85.77 | 85.88 | 86.00 | 86.11 | 86.22 | 86.33 | 86.44 | 86.55 | 86.66 | 86.77 | 86.88 | 87.00 | 87.11 | 87.22 | 87.33 | 87.44 | 87.55 | 87.66 | 87.77 | 87.88 | 88.00 | 88.11 | 88.22 | 88.33 | 88.44 | 88.55 | 88.66 | 88.77 | 88.88 | 89.00 | 89.11 | 89.22 | 89.33 | 89.44 | 89.55 | 89.66 | 89.77 | 89.88 | 90.00 | 90.11 | 90.22 | 90.33 | 90.44 | 90.55 | 90.66 | 90.77 | 90.88 | 91.00 | 91.11 | 91.22 | 91.33 | 91.44 | 91.55 | 91.66 | 91.77 | 91.88 | 92.00 | 92.11 | 92.22 | 92.33 | 92.44 | 92.55 | 92.66 | 92.77 | 92.88 | 93.00 | 93.11 | 93.22 | 93.33 | 93.44 | 93.55 | 93.66 | 93.77 | 93.88 | 94.00 | 94.11 | 94.22 | 94.33 | 94.44 | 94.55 | 94.66 | 94.77 | 94.88 | 95.00 | 95.11 | 95.22 | 95.33 | 95.44 | 95.55 | 95.66 | 95.77 | 95.88 | 96.00 | 96.11 | 96.22 | 96.33 | 96.44 | 96.55 | 96.66 | 96.77 | 96.88 | 97.00 | 97.11 | 97.22 | 97.33 | 97.44 | 97.55 | 97.66 | 97.77 | 97.88 | 98.00 | 98.11 | 98.22 | 98.33 | 98.44 | 98.55 | 98.66 | 98.77 | 98.88 | 99.00 | 99.11 | 99.22 | 99.33 | 99.44 | 99.55 | 99.66 | 99.77 | 99.88 | 100.00 | 100.11 | 100.22 | 100.33 | 100.44 | 100.55 | 100.66 | 100.77 | 100.88 | 101.00 | 101.11 | 101.22 | 101.33 | 101.44 | 101.55 | 101.66 | 101.77 | 101.88 | 102.00 | 102.11 | 102.22 | 102.33 | 102.44 | 102.55 | 102.66 | 102.77 | 102.88 | 103.00 | 103.11 | 103.22 | 103.33 | 103.44 | 103.55 | 103.66 | 103.77 | 103.88 | 104.00 | 104.11 | 104.22 | 104.33 | 104.44 | 104.55 | 104.66 | 104.77 | 104.88 | 105.00 | 105.11 | 105.22 | 105.33 | 105.44 | 105.55 | 105.66 | 105.77 | 105.88 | 106.00 | 106.11 | 106.22 | 106.33 | 106.44 | 106.55 | 106.66 | 106.77 | 106.88 | 107.00 | 107.11 | 107.22 | 107.33 | 107.44 | 107.55 | 107.66 | 107.77 | 107.88 | 108.00 | 108.11 | 108.22 | 108.33 | 108.44 | 108.55 | 108.66 | 108.77 | 108.88 | 109.00 | 109.11 | 109.22 | 109.33 | 109.44 | 109.55 | 109.66 | 109.77 | 109.88 | 110.00 | 110.11 | 110.22 | 110.33 | 110.44 | 110.55 | 110.66 | 110.77 | 110.88 | 111.00 | 111.11 | 111.22 |  |